45. $\hat{r}(t) = \left\langle \frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2} \right\rangle \rightarrow \langle -1, 0 \rangle$ ($\lim_{t\to\infty}$)

$$\hat{v}(t) = \left\langle \frac{(1+t^2)(-2t)-(1-t^2)(2t)}{(1+t^2)^2}, \frac{(1+t^2)2-(2t)(2t)}{(1+t^2)^2} \right\rangle$$

$$= \left\langle \frac{-2t-2t^3-2t+2t^3}{(1+t^2)^2}, \frac{2+2t^2-4t^2}{(1+t^2)^2} \right\rangle$$

$$= \left\langle \frac{-4t}{(1+t^2)^2}, \frac{2-2t^2}{(1+t^2)^2} \right\rangle$$

b) at rest: $|\hat{v}| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 0$

never at rest. no value of t that makes both $\frac{dx}{dt}$ & $\frac{dy}{dt} = 0$

c) $\lim_{t\to\infty} \hat{r}(t)$

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37. $\hat{v} = \langle 3t^2-2t, 1+\cos(\pi t) \rangle$ $(2, 6)$ ← position at $t=0$

a) position at $t=3$: $\left\langle \int_0^3 3t^2-2t\,dt+2, \int_0^3 1+\cos(\pi t)\,dt + 6 \right.$

$$\int 3t^2-2t\ dt$$

$t^3 - t^2 + c = 2$ $t=0$

$c = 2$

$r = t^3 - t^2 + 2 \Big|_{t=3}$

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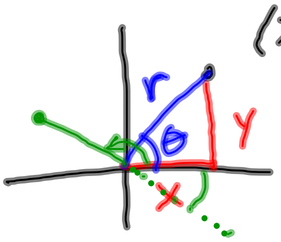
10.3 polar coordinates



(x,y)

(r,θ)

$x = r\cos\theta$

$y = r\sin\theta$

$r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$ { add π if the point (x,y) is in QII or QIII }

convert (−3,4) to polar coords

$r = 5$

$\theta = 126.87^\circ$ $-.927$

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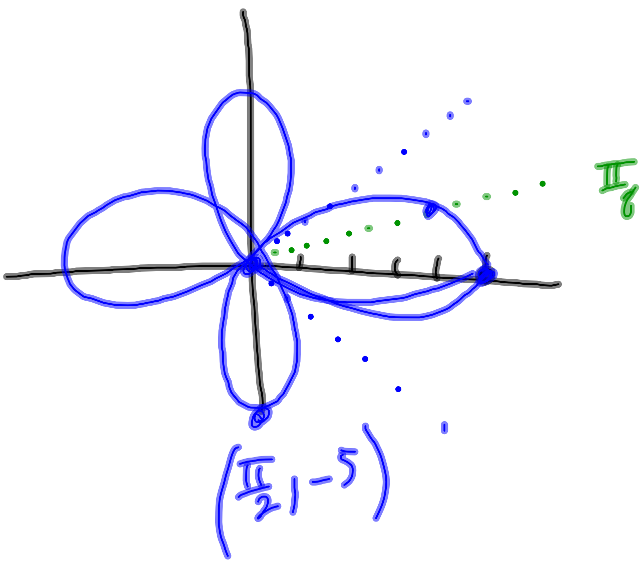
Polar functions $r = f(\theta)$

$r = 5\cos(2\theta)$

θ	r
0	5
$\frac{\pi}{4}$	0
$\frac{\pi}{8}$	$5\frac{\sqrt{2}}{2} \approx 3.5$
$-\frac{\pi}{4}$	0
$\frac{\pi}{2}$	−5



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slope of a polar graph $r = f(\theta)$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$x = r\cos\theta = f(\theta)\cos\theta$$

$$y = r\sin\theta = f(\theta)\sin\theta$$

$r = 5\cos 2\theta$

$$x = 5\cos(2\theta)\cos\theta$$

$$y = 5\cos(2\theta)\sin\theta$$

$$\frac{dy}{dx} = \frac{5\cos(2\theta)\cdot\cos\theta + \sin\theta\cdot(-10\sin(2\theta))}{5\cos(2\theta)(-\sin\theta) + \cos\theta(-10\sin(2\theta))}$$

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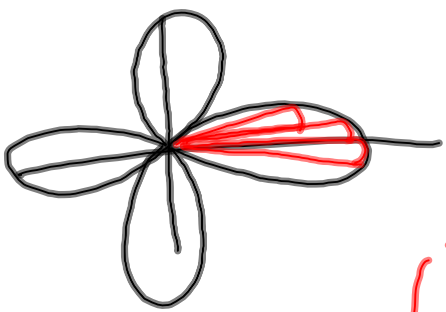
area



sector

$$\text{area} = \frac{1}{2}r^2\Delta\theta$$

$$\lim_{\Delta\theta \to 0} \sum \frac{1}{2} r_i^2 \Delta\theta = \int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 \, d\theta$$

area inside the polar graph

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Find the area inside 1 leaf of

$r = 5\cos(2\theta)$

$$\int_{-\pi/4}^{\pi/4} \frac{1}{2}\left(5\cos(2\theta)\right)^2 d\theta$$

or $2\int_0^{\pi/4}$ or $\int_0^{\pi/2}$

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area between 2 polar graphs

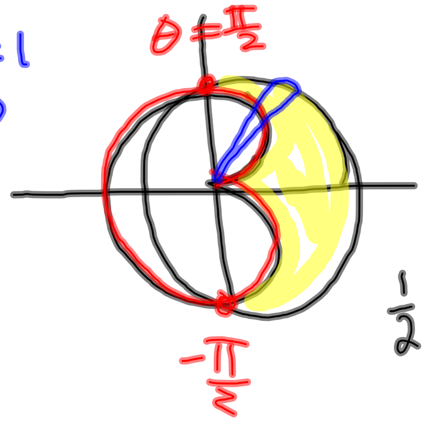
area outside the cardioid $r = 1 - \cos\theta$

and inside the circle $r = 1$

$1 - \cos\theta = 1$

$\cos\theta = 0$

$\theta = \pm\frac{\pi}{2}$



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$$\int_{\theta_1}^{\theta_2} \frac{1}{2}R^2 - \frac{1}{2}r^2 d\theta$$

$$\frac{1}{2}\int_{-\pi/2}^{\pi/2} 1^2 - (1-\cos\theta)^2 d\theta$$

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